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Uncertainty and specification challenges in exchange rates modeling under Bayesian model averaging

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Abstract. Traditional econometric models often struggle to capture the complexities and uncertainties inherent in exchange rate movements. This study investigates the dynamic relationship between the Nigerian Naira exchange rate and key economic variables using Bayesian Model Averaging (BMA), offering a robust framework to address model uncertainty and specification challenges. By integrating multiple predictors, BMA enhances forecasting accuracy, providing valuable insights for policymakers, investors and businesses navigating Nigeria's volatile economic landscape. Analysing exchange rate movements from 2007 to 2021, the study assesses BMA's effectiveness compared to traditional models, particularly in capturing non-linear relationships and time-varying volatility. Findings reveal that capital expenditure and the money supply are the most significant determinants of exchange rate fluctuations: capital expenditure negatively affects the exchange rate, while increased money supply leads to currency appreciation. These results highlight the potential of BMA to refine economic forecasts and improve decision-making in Nigeria's financial and policy sectors.

Keywords: Bayesian model averaging, exchange rate, forecasting, Nigerian Naira, structural breaks, volatility clustering.

2020 Mathematics Subject Classification: 62C10, 62C12, 62P25.

1 Introduction

In the contemporary era, characterized by macroeconomic volatility across nations, exchange rates serve as a fundamental indicator of the value of one currency relative to another. Structurally, an exchange rate comprises two primary elements: the domestic (internal) currency and an international currency. Exchange rate quotations are typically classified as either direct or indirect. A direct quote indicates the amount of domestic currency required to purchase 1 unit of foreign currency. Moreover, exchange rates can be expressed as cross-currency rates or

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cross rates. As a pivotal variable in international finance, [2] identified the variables that significantly influence exchange rate volatility by quantitatively measuring their determinants. In recent decades, [13] compared ANN and ARIMA models' predictive capacity as applicable to accurate forecasting of stock prices using training data. The study asserted that ANN is greatly preferred to ARIMA in terms of accuracy.

The foreign exchange (FX) market has experienced significant expansion, with exchange rates emerging as a fundamental determinant of market behavior and volatility. Despite this growth, accurately modeling the stochastic behavior of exchange rate movements remains a complex endeavour, mainly due to the multifaceted and dynamic nature of the influencing macroeconomic and geopolitical variables.
Conventional econometric frameworks, such as linear regression and time series models, often struggle to accommodate the high degree of model uncertainty and structural instability characteristic of exchange rate data.

To address these limitations, BMA offers a robust alternative by incorporating model uncertainty into the inferential process. BMA systematically evaluates a set of competing models, estimating the posterior probability of each model given the observed data. Rather than relying on a single model specification, BMA constructs a weighted average of model predictions, with weights derived from the respective posterior probabilities.
This probabilistic framework allows for improved predictive performance by integrating the information from multiple models, thereby capturing a broader spectrum of potential explanatory variables and structural relationships. This methodological framework facilitates enhanced forecast accuracy and provides deeper insights into the underlying determinants of exchange rate dynamics. In the Nigerian context, the Naira's exchange rate relative to major international currencies is highly volatile, posing persistent challenges for monetary authorities, institutional investors, and corporate entities. Despite concerted efforts to model and forecast exchange rate movements, traditional econometric techniques have demonstrated limited efficacy in capturing the inherent model uncertainty, parameter instability, and nonlinear interactions among explanatory variables.

A fundamental limitation of these conventional models is their reliance on a static set of covariates, which often fails to reflect the evolving nature of the macroeconomic environment. Additionally, structural breaks, such as shifts in monetary policy, external shocks, or changes in trade regimes, often compromise the predictive validity of these models. Consequently, these shortcomings undermine the robustness of exchange rate forecasts, adversely affecting strategic decision-making and the implementation of effective risk mitigation strategies in both public and private sectors. Exchange rate dynamics are inherently time-varying, influenced by shifts in macroeconomic fundamentals, monetary and fiscal policy interventions, and exogenous global shocks. As such, the assumption of temporal stability embedded in many conventional static models often proves inadequate. These models typically overlook parameter instability and structural evolution, leading to rapidly obsolete forecasts during periods of regime change or market realignment. Neglecting such dynamic behavior can result in mis-specified models, flawed empirical inferences, and suboptimal policy interventions, thereby amplifying macroeconomic volatility and deepening systemic uncertainty.

The conceptual framework underpinning this study integrates theoretical constructs and methodological principles drawn from exchange rate theory, Bayesian inference, and moving average techniques. This framework serves as a foundation for exploring the application of BMA in forecasting exchange rate behavior, particularly in the Nigerian context. It highlights the synergies among these key components and delineates their relevance to the research objectives. Exchange rate modeling involves projecting future currency values based on historical trends, macroeconomic indicators, and market expectations. Generating reliable forecasts is critical for policy formulation, investment strategies, and risk management. Bayesian approaches provide a probabilistic modeling paradigm, enabling the incorporation of prior distributions and the continuous refinement of forecasts as new information becomes available. Meanwhile, moving average techniques are employed to smooth high-frequency fluctuations in time-series data, thereby isolating persistent long-term trends and enhancing signal extraction in noisy datasets.

Bayesian inference is a foundational statistical framework that combines prior distributions reflecting existing knowledge or expert beliefs with empirical data to derive posterior distributions of unknown parameters. This approach supports a dynamic learning process, wherein model parameters are continually updated as new data become available, thereby enhancing predictive accuracy and informing evidence-based decision-making [4]. The Bayesian paradigm is particularly advantageous in contexts characterized by uncertainty and limited data, offering a coherent mechanism for probabilistic reasoning. Time series analysis constitutes a class of statistical techniques tailored for analyzing temporally ordered observations. Central to this domain is the Box-Jenkins methodology, which introduced the Auto Regressive Integrated Moving Average (ARIMA) model, a versatile tool for modeling and forecasting univariate time series. [3] showed that ARIMA models are especially effective in capturing trend, seasonality, and autocorrelation structures in exchange rate data, thereby facilitating the development of statistically robust forecasting systems. Bayesian time series models enhance traditional time series frameworks by embedding Bayesian inference, thereby enabling more flexible and adaptive modeling under conditions of uncertainty. The seminal work of [17] in 1997 on Dynamic Linear Models (DLMs) and state-space formulations provides a rigorous framework for modeling evolving time-series processes. These models are particularly well-suited for financial and economic applications, including exchange rate forecasting, due to their ability to accommodate latent states, integrate prior knowledge, and adapt to structural changes in the underlying data-generating process. The Kalman Filter [12] is a recursive computational algorithm that produces optimal estimates of latent variables by assimilating sequential, noisy measurements. It has become an integral component of Bayesian time series analysis, especially in state-space models, where it facilitates signal extraction from stochastic noise, thereby enhancing the forecast precision of exchange rate models.

In the Nigerian context, [1] investigated the dynamic nexus between exchange rate fluctuations and key macroeconomic indicators using monthly time series data spanning 2000-2015, obtained from national financial databases. The study employed a Vector Autoregressive (VAR) framework, with optimal lag lengths selected via Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC). The findings revealed that inflation and interest rates were significant determinants of exchange rate variability, as corroborated by impulse response functions (IRFs) and variance decomposition analysis. The authors in [11] compared traditional time-series techniques with Bayesian forecasting methods using quarterly exchange rate data (2005-2017). Their study demonstrated that Bayesian approaches, particularly those leveraging Gibbs sampling, outperformed classical ARIMA models, especially under high-volatility regimes. Similarly, Bayesian techniques were applied in [6] to high-frequency exchange rate data from 2010 to 2019, incorporating BMA within a state-space modeling framework and employing Kalman filtering to enhance inference. The results confirmed the superior robustness and predictive accuracy of Bayesian models, which effectively captured uncertainty in the posterior distributions. The usefulness of BMA in the presence of structural breaks was investigated, in [18], through the modeling of exchange rate time series from 1995 to 2018, incorporating dynamic model selection and out-of-sample forecasting procedures. Their empirical findings validated BMA's superior predictive performance over traditional single-model strategies, underscoring its ability to account for model uncertainty. Bayesian Vector Autoregressive (BVAR) models were assessed using quarterly data spanning 2000–2019, integrating stochastic volatility and Markov Chain Monte Carlo (MCMC) methods, as reported in [16]. The results demonstrated that BVAR models produced more reliable forecasts, particularly during periods of macroeconomic instability. Furthermore, Bayesian Structural Time Series (BSTS) models were evaluated for forecasting financial trends, demonstrating resilience during crisis periods through the incorporation of local trend components and external shocks, with parameter estimation conducted via Gibbs sampling [14]. The literature was further extended by combining Bayesian inference with machine learning, specifically through the development of a Bayesian Neural Network (BNN) trained using variational inference on monthly data from 2000 to 2021 [15]. Their model adeptly captured nonlinear dependencies in the exchange rate series, outperforming conventional Bayesian techniques. A synthesis of these empirical studies reveals persistent gaps in the literature, particularly the under utilization of BMA techniques in exchange rate forecasting, despite mounting evidence of their effectiveness in handling model misspecification and forecast uncertainty. Most existing work remains anchored in classical filtering techniques or limited applications of Bayesian models, overlooking BMA's distinct advantage of integrating multiple model structures to enhance forecast accuracy in volatile markets like Nigeria's. This study applies BMA as an advanced framework for modeling and forecasting exchange rate dynamics in Nigeria. It explores the time-varying relationship between the Naira and key macroeconomic variables to enhance predictive accuracy. The research evaluates BMA's effectiveness in capturing volatility clustering, identifying structural breaks, and mitigating model uncertainty and specification bias. Additionally, it examines the macroeconomic implications of exchange rate fluctuations for external competitiveness, inflation, and monetary policy. The study ultimately aims to generate evidence-based insights for policymakers, investors, and financial analysts involved in exchange rate risk management and strategic economic planning.

2 Materials and methods

2.1 Linear model for exchange rate

Consider a situation where, in each year, the exchange rate is observed on n_i occasions. Let Y_i be the vector of responses to the exchange rate for the i-th year, arising from the linear model:

$$Y_i = X_i\beta + \epsilon_i, \tag{2.1}$$

where X_i is the design matrix for the i-th exchange rate, and $\epsilon_i \sim MVN(0, \Sigma)$ is a vector of deviations with a multivariate normal distribution and an unspecified covariance structure. When ϵ_i is independent of X_i, the exchange rate data are said to be completely balanced. In the presence of fluctuating exchange rates or missing observations, autoregressive (AR) models are adopted to provide a more parsimonious structure for modeling the inflation rate.

2.2 Bayesian model averaging framework

BMA provides a principled approach to addressing model uncertainty, especially in regression contexts where the choice of explanatory variables is ambiguous. Instead of selecting a single "best" model, BMA averages over the entire model space, weighting models by their posterior probabilities. This yields inferences that are more robust to specification errors and allows for the identification of covariates that are truly relevant to the underlying data-generating process, conditional on the prior structures [9].

Consider the standard linear regression framework with an intercept and k candidate predictors $X_1, X_2, \ldots, X_k$. The model can be expressed as:

$$y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_k X_k + \varepsilon, \tag{2.2}$$

where y denotes the response variable, $\beta_0, \beta_1, \ldots, \beta_k$ are unknown regression coefficients, and ε is the stochastic error term assumed to be normally distributed with mean zero and constant variance.

This formulation gives rise to a total of 2^k distinct models (denoted M_j, for $j = 1, 2, 3, \ldots, 2^k$), depending on the inclusion or exclusion of each of the k potential regressors [8]. After specifying the model space, BMA combines these models to estimate the posterior distribution of parameters of interest. For any parameter θ, the model-averaged posterior distribution given the observed data D is expressed as:

$$P(\theta \mid D) = \sum_{j=1}^{2^k} P(\theta \mid M_j, D) \cdot P(M_j \mid D), \tag{2.3}$$

where, $P(M_j \mid D)$ represents the posterior model probability of model M_j, and $P(\theta \mid M_j, D)$ is the posterior distribution of θ under model M_j [5].

This formulation ensures that inference and prediction are informed not only by the most probable model but by all plausible models, proportionally weighted by how well they explain the observed data. Thus, BMA serves as a powerful technique in empirical research where model uncertainty poses a significant challenge.

2.2.1 Posterior inclusion probabilities

The posterior inclusion probability of each coefficient of interest β_h, given the data D, is a measure of how likely it is that the coefficient is included in the true model. The posterior distribution of β_h is given by:

$$P(\beta_h \mid D) = \sum_{i=1}^{2^k} P(\beta_h \mid M_j) P(M_j \mid D), \tag{2.4}$$

where:
i. P($\beta_h|D$) is the posterior probability of β_h given the data.
ii. M_j represents different models considered.
iii. P($\beta_h|M_j$) is the posterior probability of β_h given model M_j
iv. P($M_j \mid D$) is the posterior model probability.

2.2.2 Posterior model probabilities

The posterior model probability, P$M_j \mid D$ is the probability that model M_j is the true model given the data D. It is calculated as the ratio of the marginal likelihood of the model to the sum of the marginal likelihoods of all considered models:

$$P(M_j|D) = \frac{P(M_j \mid D) P(M_j)}{P(D)}. \tag{2.5}$$

This can also be expressed as:

$$P(M_j \mid D) = \frac{P(D \mid M_j)P(M_j)}{\sum_{i=1}^{2^k} P(D \mid M_j)P(M_j)}, \tag{2.6}$$

where:
i. $P(D \mid M_j)$ is the marginal likelihood of model M_j.
ii. $P(M_j)$ is the prior probability of model M_j.
iii. P(D) is the probability of the data under all models.

2.2.3 The integrated likelihood

The integrated likelihood (also known as the marginal likelihood) is the probability density of the data, conditional on the model M_j. It integrates the likelihood over the parameter space, weighted by the prior distribution:

$$P(D \mid M_j) = \int P(D)\beta_h, M_j)P(\beta_h \mid M_j)d\beta_h, \tag{2.7}$$

where:
i. β_h is the vector of parameters for model M_j
ii. $P(D|\beta_h, M_j)$ is the likelihood of the data given parameters β_h and model M_j
iii. $P(\beta_h \mid M_j)$ is the prior distribution of β_h under model M_j.

2.2.4 Priors specification

Assigning prior distributions on both the parameters and the model space is a crucial step in BMA. [7] emphasized the challenges in this aspect.

2.2.5 Assigning model priors

In BMA, non-informative priors can lead to improper predictive distributions. To avoid this, we use uniform priors for model probabilities:

$$P(M_j) = \frac{1}{2^k}, \tag{2.8}$$

where

$$\sum_{i=1}^{2^k} P(M_j) = 1.$$

According to [10], the Unit Information Prior (UIP) generally outperforms other priors in linear regression models. Hence, UIP is used for this study.

2.2.6 Posterior mean and variance

The estimated means and variance of $\hat{\beta}$=($\hat{\beta}_0$, $\hat{\beta}_1$,$\cdots$,$\hat{\beta}_k$) are then constructed as:

$$E(\hat{\beta} \mid D) = \sum_{j=1}^{2^k} \hat{\beta}P(M_j \mid D), \tag{2.9}$$

$$V(\hat{\beta} \mid D) = \sum_{j=1}^{2^k} (var[\hat{\beta} \mid D, M_j] + \hat{\beta}^2 P(M_j \mid D)) - E(\hat{\beta} \mid D)^2, \tag{2.10}$$

where,

$$\hat{\beta}_i = E(\hat{\beta} \mid D, M_j).$$

2.2.7 BMA predictive performance

BMA is renowned for its ability to enhance predictions, as demonstrated by out-of-sample prediction error. [10] provided several examples showcasing BMA's predictive capabilities through cross-validation.

2.2.8 Log predictive performance

The predictive performance of a single model M is measured using the logarithmic scoring rule:

$$-\sum_{\beta \in D^p} \log \left\{ P(\beta \mid M, D^T) \right\}. \tag{2.11}$$

For BMA, the predictive ability is measured by:

$$-\sum_{\beta \in D^p} \log \left\{ \sum_{i=1}^{I} P(\beta \mid M_i, D^T) \cdot P(M_i \mid D^T) \right\}. \tag{2.12}$$

3 Results and discussion

3.1 Data presentation

The annual official exchange rate of the Nigerian Naira spanning the period 2007 to 2021 was sourced from the Statistical Bulletin of the Central Bank of Nigeria (CBN) [https://www.cbn.gov.ng], and the World Development Indicators (WDI) database maintained by the World Bank. In the present analysis, the exchange rate (EXR) serves as the dependent (endogenous) variable, while the following indicators are treated as independent (exogenous) variables: Stock Exchange Capital Expenditure (CE), Imports (IMP), Exports (EXP), Interest Rate (IR), Inflation Rate (INF), Money Demand (MD), Money Supply (MS), Balance of Trade (BOT), Real Effective Exchange Rate (REER), Net Income from Abroad (NIFA), Crop Production (CP), Foreign Direct Investment (FDI), and Interest Payments on External Debt (IPEDT). These variables were selected based on their theoretical and empirical relevance to exchange rate determination in macroeconomic modeling frameworks.

Table 3.1 presents summary statistics for the five-number summary, mean, standard deviation, and coefficient of variation to assess the variability of the key economic variables. It is observed that Balance of Trade (BOT), Money Demand (MD), and Foreign Direct Investment (FDI) are among the economic variables with high variability.

Table 3.1: Summary statistics of Naira exchange rates and key economic variables

Variables	Max	Min	Q1	Q3	Q2	Mean	Std. Dev.	CV 100%
EXR	411.98	118.57	152.09	305.94	158.55	220.42	94.64	42.93
CE	2214.20	842.50	1353.05	2062.60	1735.1	1669.79	447.61	26.81
CP	118.31	72.71	85.81	109.76	97.61	96.80	14.75	15.23
Export	146.37	35.09	51.04	94.03	65.36	75.14	33.59	44.70
Import	72515.00	35404.00	48078.50	63201.50	53670	55314.87	11822.51	21.37
FDI	8940.20	1634.80	2864.95	7523.15	4491.1	4989.69	2702.86	54.17
IPOD	1169.10	411.40	691.40	837.55	737.2	779.62	201.04	25.79
INF R	0.17	0.05	0.10	0.14	0.122	0.12	0.03	27.57
MD	31811.60	4470.60	8736.15	20407.00	14086.4	15316.40	8301.81	54.20
MS	34097224.70	7381641.70	12771734.15	24217531.35	18045248	18919182.91	7997273.30	42.27
NIFA	25210.12	8698.89	13739.36	18867.93	15772.71	16449.77	4518.35	27.47
RE	124.20	92.40	99.65	113.50	101.1	105.97	9.53	8.99
SE	57990.22	20730.63	26858.35	39256.96	31430.51	32990.28	9860.98	29.89
BOT	27562.50	-5053.10	-655.80	11367.25	2911.5	5848.71	9235.46	157.91
INT.R	0.14	0.06	0.10	0.13	0.12	0.11	0.03	22.96

3.2 BMA analysis

BMA is employed to address model uncertainty by integrating inference over a set of candidate models. Each model is assigned a posterior probability reflecting its plausibility given the observed data and prior information. Parameter estimates are obtained as weighted averages across all considered models, thereby formally accounting for model selection uncertainty and enhancing robustness.

Table 3.2 summarizes the procedural steps and results associated with the implementation of BMA in the present study.

Table 3.2: Summary of output from Bayesian model averaging analysis

Mean no. regressors 4.3676	Draws 50000	Burnins 10000
Time (secs) 9.213531	No. models visited 28657	Model space 8192
% visited 350	Top models (%) 90	Corr PMP 0.9855
Model Prior & Uniform 6.5	g-Prior UIP	Shrinkage-Stats (Av) 0.9375

The analysis revealed that the posterior mean number of covariates included per model was approximately 4.37, indicating consistent selection of four to five statistically significant predictors in modeling exchange rate behavior. This result was obtained from a MCMC sampling scheme executed over 50,000 iterations, with the first 10,000 samples discarded as part of a burn-in process to ensure convergence and eliminate dependence on initial values.

The burn-in phase stabilized posterior estimates by removing transients arising from the initial parameter configurations. The BMA algorithm demonstrated notable computational efficiency, completing its execution in approximately 9.21 seconds.

This performance reflects the use of advanced optimization routines that can traverse a high-dimensional model space with minimal computational overhead. Of the 8,192 theoretically possible model combinations derived from 13 potential explanatory variables (i.e., 2^{13}), a total of 28,657 unique models were actually sampled and evaluated. This number represents an exhaustive model exploration strategy that is approximately 350% of the theoretical model space, thereby enhancing the reliability of model selection.

A cumulative posterior probability threshold was used to select 90% of the most probable models for summary, thereby emphasizing models with strong empirical support. The computed posterior model probability correlation (Corr PMP) was 0.9855, signifying high internal coherence and stability among the dominant models.

A uniform prior distribution was assumed over the model space, with a prior inclusion probability of 6.5% per covariate, and regression coefficients were assigned the Unit Information Prior (UIP), a type of Zellner's g-prior that achieves a balance between model parsimony and explanatory adequacy.

The average shrinkage factor was estimated at 0.9375, indicating substantial regularization of the regression coefficients to prevent overfitting and favor predictors with high posterior relevance.

Table 3.3: Posterior model probabilities of some selected top models

M	CE	CP	Ex	Im	FDI	IPOD	IR	MD	MPR	MS	NIFA	RE	SE	PMP(E)	PMP(M)
1	1	0	0	0	0	0	0	0	0	1	0	0	0	0.030635	0.03222
2	1	0	0	0	0	0	0	1	0	0	0	0	0	0.012497	0.0137
3	1	0	0	0	0	0	0	0	0	1	0	0	1	0.010645	0.01068
4	1	0	0	0	0	1	0	0	0	1	0	0	0	0.010057	0.01258
5	1	0	0	1	0	0	0	0	0	1	0	0	0	0.00923	0.00802

Note: M = Model, Ex = Export, Im = Import, PMP(E) = PMP (Exact), PMP(M) = PMP (MCMC)

Table 3.3 represents the posterior model probabilities (PMP) and the inclusion status of various regressors across different model indices. The values indicate whether a regressor is included (1) or not (0) in the specified models, along with the exact and MCMC estimated PMPs for each model.

Table 3.4: Posterior model probabilities of the five top models

Model	PMP (Exact)	PMP (MCMC)
1008	0.030635	0.03222
1020	0.012497	0.0137
1009	0.010645	0.01068
1088	0.010057	0.01258
1208	0.00923	0.00802

Table 3.4 presents a comparative summary of the PMPs associated with the five most plausible models, reported using both analytical computations and MCMC approximations. Among the evaluated specifications, Model 1008 emerges as the most probable candidate, exhibiting an exact PMP of 0.030635 and a corresponding MCMC-based estimate of 0.03222. This strong alignment between analytical and simulation-based methods suggests a high degree of confidence in selecting Model 1008 as the best-fitting structure given the data.

Subsequent models exhibit decreasing levels of support. Model 1020 records PMPs of 0.012497 (exact) and 0.0137 (MCMC), positioning it as a prominent alternative, albeit with a lower posterior weight than Model 1008. Model 1009 follows closely, with exact and MCMC-derived PMPs of 0.010645 and 0.01068, respectively, indicating a comparable but slightly diminished likelihood. Model 1088 yields PMPs of 0.010057 (exact) and 0.01258 (MCMC), maintaining its presence among the top contenders despite lower posterior support. Lastly, Model 1208 registers the weakest probabilities in the group, with exact and MCMC PMPs of 0.00923 and 0.00802, respectively, making it the least supported among the leading configurations.

Collectively, the PMP values underscore Model 1008 as the dominant posterior candidate, while the remaining models 1020, 1009, 1088, and 1208 retain relevance as alternative structures with descending levels of plausibility. The high concordance between the exact and MCMC estimates across all five models reinforces the robustness and consistency of the posterior inference process.

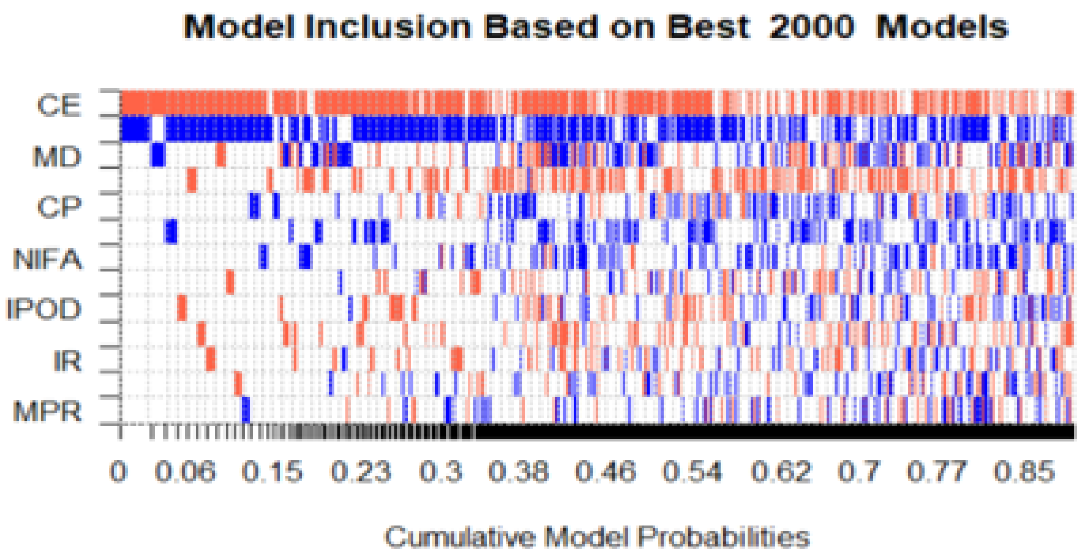

Figure 3.1: Chart showing cumulative probabilities on the best 2000 model

To gain deeper insights into model performance and variable relevance, we examine the graphical output of model inclusion probabilities. Figure 3.1 illustrates the Cumulative Inclusion Probabilities (CIPs) across the leading models, with visual encoding based on coefficient signs: blue indicates a positive estimated coefficient, red represents an adverse effect, and white denotes exclusion from the model (i.e., a zero coefficient). The horizontal axis ranks models in descending order of their PMPs. This visual summary clearly identifies Capital Expenditure (CE) as the most influential predictor, consistently included in top models and positively associated with the exchange rate. This is evidenced by the dominant blue color in its row, signifying that increases in capital expenditure are associated with a corresponding rise in the exchange rate. Money

Supply (MS) follows closely and also exhibits a positive relationship, as shown by the blue segments, further indicating its substantial contribution. Conversely, variables that are not consistently selected in the top-performing models are represented by white cells, indicating low posterior inclusion probabilities and suggesting minimal influence on the exchange rate within the Bayesian model framework.

Figure 3.2 shows that the model prior is symmetrically distributed around $\frac{k}{2} = 6.5$. Updating this prior with the data yields a posterior distribution that places greater weight on models of intermediate size. Consequently, the posterior model size distribution remains close to the prior.

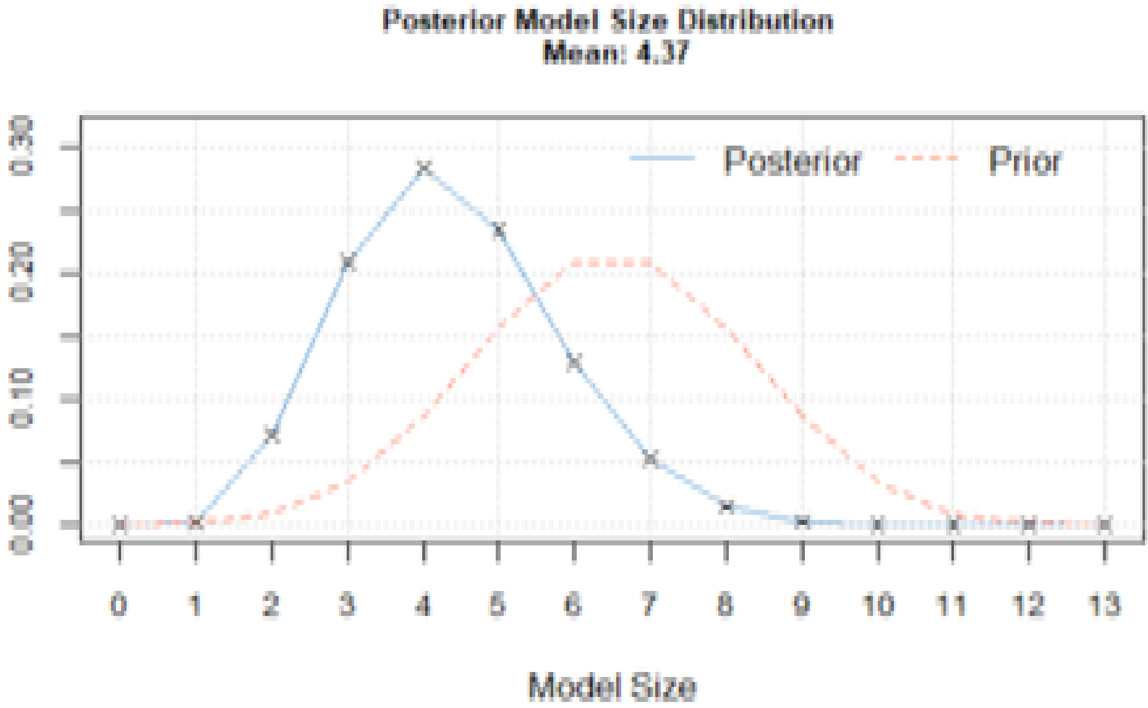

Figure 3.2: Posterior model size distribution with uniform prior

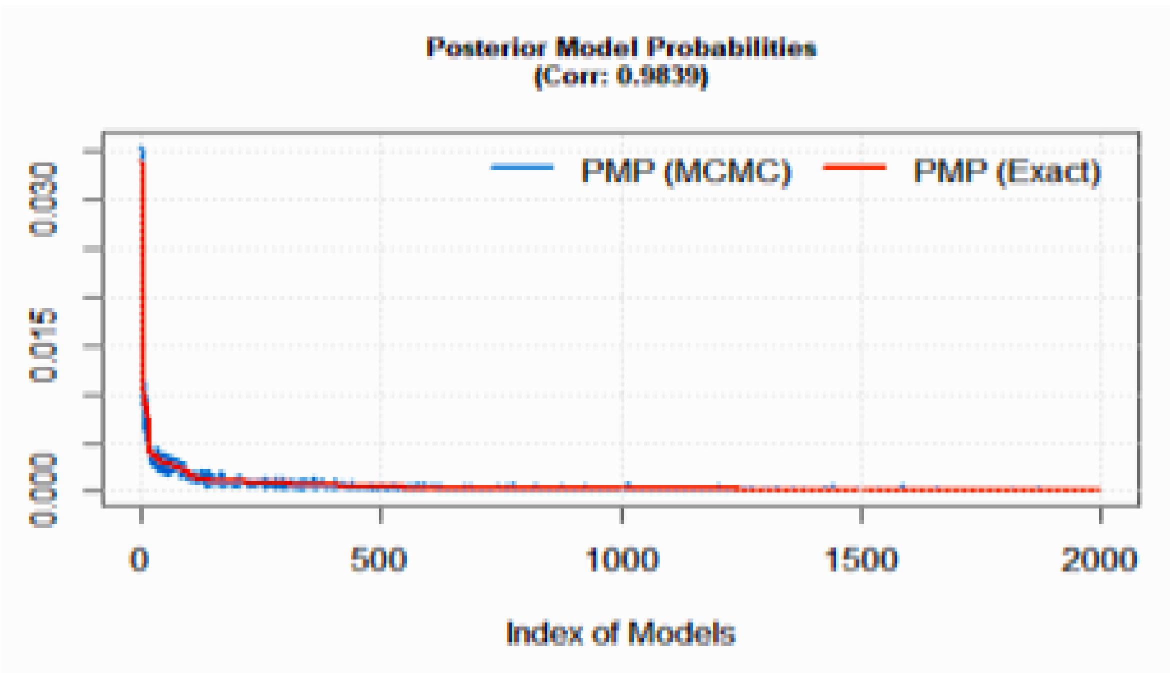

Figure 3.3: Posterior model probabilities with uniform prior

Figure 3.3 presents a comparative visualization of the top 2,000 models ranked by their analytical PMPs, represented by the red curve, alongside the corresponding selection frequencies in the MCMC sampling process, indicated by the blue line. When the distribution of marginal likelihoods is highly complex, achieving convergence to accurate PMP estimates becomes more challenging. The fidelity of the MCMC approximation can be evaluated by comparing the frequency with which each model is sampled to its analytically derived marginal likelihood. The observed alignment between the sampling frequencies and analytical PMPs, culminating in a convergence value of approximately 0.9839, reflecting a substantial degree of coherence, supports the reliability of the model space exploration achieved through MCMC. Building on the BMA results, Table 3.5 provides a summary of the posterior statistics for each explanatory variable in the regression model, with the exchange rate as the response variable. The table includes key indicators for each predictor, including the Posterior Inclusion Probability (PIP), posterior mean, and posterior standard deviation. These measures collectively indicate the relative importance and direction of influence of each regressor within the Bayesian framework.

Table 3.5: Posterior probabilities for each regressor

Regressor	PIP	Posterior mean	Posterior Std. Dev.
Capital expenditure (NGN)	0.73326	-0.58479	0.46691
Money supply	0.6793	1.103564	1.245474
Money demand (NGN)	0.39206	-0.07026	0.906892
Import (USD)	0.36164	-0.17794	0.367961
Crop production	0.2938	0.357028	1.447852
Stock exchange (Points)	0.27928	0.033007	0.10051
Net income from abroad (USD)	0.25194	0.035693	0.171355
Foreign direct investment (USD)	0.24568	0.004745	0.079154
Real-EFF	0.2389	-0.07376	0.547721
Export (USD)	0.2311	-0.01086	0.108402
Inflation rate	0.22964	-0.00527	0.097141
Interest paid on external debt (USD)	0.22344	0.006574	0.29975
Interest (MPR)	0.21992	3.35e-05	0.110298

Capital expenditure (NGN) has a high PIP of 0.73326, indicating it is a likely significant factor. The posterior mean of -0.58479 suggests a negative relationship with the exchange rate, implying that increases in capital expenditure might lead to currency depreciation. Money supply is another key variable, with a PIP of 0.6793. Its posterior mean is 1.103564, suggesting a positive relationship with the exchange rate, in which increases in the money supply may lead to currency appreciation. Money demand (NGN) has a PIP of 0.39206, suggesting a moderate likelihood of inclusion in the model. The posterior mean is -0.07026, suggesting a slight adverse effect on the exchange rate. Similarly, imports (Dollars) have a PIP of 0.36164 and a posterior mean of -0.17794, indicating an adverse effect. Crop production shows a lower PIP of 0.2938 and a positive posterior mean of 0.357028. Stock exchange (points) has a PIP of 0.27928, with a small positive posterior mean of 0.033007 and a low standard deviation of 0.10051. Net income from abroad (Dollars) and foreign direct investment (USD) have PIPs of 0.25194 and 0.24568, respectively, with small positive posterior means of 0.035693 and 0.004745. Their effects are relatively specific, as indicated by their standard deviations of 0.171355 and 0.079154, respectively. Real effective exchange rate (RE) has a PIP of 0.2389 and a negative posterior mean of -0.07376, with a standard deviation of 0.547721, indicating moderate uncertainty. Exports (billions of USD) show a PIP of 0.2311, a very small negative posterior mean of -0.01086, and a low standard deviation of 0.108402, suggesting a relatively small adverse effect on the exchange rate. The inflation rate has a PIP of 0.22964, a small negative posterior mean of -0.00527, and a standard deviation of 0.097141, indicating a minimal adverse effect. Interest paid on external debt (USD) has a PIP of 0.22344, with a small positive posterior mean of 0.006574 and a moderate standard deviation of 0.29975, indicating some uncertainty. Finally, the interest rate (MPR) has the lowest PIP of 0.21992, with a negligible posterior mean of 0.000033516 and a standard deviation of 0.110298, suggesting minimal impact and moderate uncertainty. Overall, capital expenditure and the money supply are the most influential factors on the exchange rate, with the former exerting an adverse effect and the latter a positive one. However, the considerable uncertainty in some estimates underscores the need for additional data or more refined models to improve the precision of these effects.

Table 3.6: Predictive values for the exchange rate

Actual	Predicted	Absolute difference
4.834773	4.791624	0.043149
4.775504	4.833202	0.057698
5.003141	4.992652	0.010489
5.012766	5.076279	0.063513
5.036043	5.018342	0.017731
5.059489	5.093882	0.034393
5.058218	5.086401	0.028183
5.06607	5.134176	0.068106
5.259784	5.358115	0.098331
5.535324	5.585900	0.050576
5.722899	5.658618	0.064283
5.723847	5.629183	0.094664
5.726587	5.673128	0.053459
5.882793	5.879005	0.003788
6.020975	5.907706	0.113269

Table 3.6 compares the actual and predicted exchange rate values. The actual values correspond to the observed exchange rate data, while the predicted values are those estimated by the model. Overall, the predicted values closely match the actual values, indicating that the model provides reasonably accurate forecasts. For example, the expected value of 4.791624 is very close to the actual value of 4.834773, reflecting a minimal prediction error. Similarly, the predicted value of 5.658618 aligns well with the actual value of 5.722899, demonstrating a strong fit at that point. However, some deviations occur in which the predicted values differ from the actual values. For instance, the expected value of 5.076279 exceeds the actual value of 5.012766, suggesting a slight overestimation. Likewise, the predicted value of 5.134176 is higher than the actual value of 5.06607, indicating another minor overestimation. Despite these variations, the overall trend suggests that the model effectively captures the patterns and fluctuations in the exchange rate. The strong alignment between many actual and predicted values underscores the model's robustness in forecasting exchange rates, though further refinements could enhance its accuracy in specific cases.

3.3 Conclusion

The application of BMA in this study identified capital expenditure and the broad money supply as the most influential macroeconomic determinants of exchange rate fluctuations in Nigeria. The analysis revealed a negative association between capital expenditure and exchange rate levels, indicating that increased public capital spending exerts depreciatory pressure on the domestic currency, while an expansion in the money supply contributes to currency appreciation. BMA effectively mitigated model uncertainty by synthesizing inferences across a suite of plausible models, thereby generating more robust and reliable exchange rate forecasts than conventional econometric methodologies. Furthermore, BMA demonstrated sensitivity to structural shifts in macroeconomic policy and fundamentals, with models incorporating these dynamic predictors achieving the highest posterior model probabilities. The strong predictive performance of BMA was validated by the close congruence between actual and estimated exchange rates, underscoring its efficacy as a reliable tool for forecasting and policy analysis. In summary, the findings affirm that Bayesian Model Averaging is a methodologically rigorous and practical approach for exchange rate modeling in the Nigerian context. Its capacity to resolve issues of model misspecification, address parameter instability, and enhance forecast precision offers significant value for macroeconomic policy formulation and strategic financial planning. In the near future, the research will be extended to the BMA framework by incorporating nonlinear predictors, global economic shocks, and machine learning-enhanced Bayesian methods to further refine forecasting accuracy.

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Appendix

Table .7: The annual official economic variables for the period of 2007-2014

Reg/year	2007	2008	2009	2010	2011	2012	2013	2014
CE	842.5	1057.2	1148.8	1140.3	1557.3	1768.1	2150.6	2214.2
CP	72.71	75.81	80.81	83.91	87.71	89.41	92.81	97.61
Export	59.09	87.14	54.96	94.17	131.04	146.37	93.88	105.85
Import	41545	55609	45354	56005	72457	72515	70398	72136
FDI	3097.4	8940.2	8077.1	8918.5	8929.1	6969.2	5515.6	4773.4
IPOD	411.4	558.4	622.2	683.7	737.2	699.1	703.4	716.1
INF R	5.40%	15.10%	12.30%	13.70%	10.80%	12.20%	8.50%	8.00%
MD	4470.6	5620.5	6662.3	7933.1	9539.2	11056.1	12731.2	14086.4
EXR	125.81	118.57	148.88	150.32	153.86	157.51	157.31	158.55
MS	7381641.7	9171151.2	10662906.5	11794080.5	13749387.8	15735734.1	16832151.1	18045248.3
NIFA	11848.37	15264.37	14718.95	19412.1	22488.06	21898.95	25210.12	18242.62
RE	98.6	96.5	92.4	100.1	99.5	101.1	100.3	99.8
SE	57990.22	31450.78	20827.17	24770.52	20730.63	28078.81	41329.19	34657.15
BOT	15804.5	27562.5	8579.1	13796.2	14598.1	6488.5	1329.5	-4938.1
INT.R	8.75%	9.75%	6%	6.25%	12%	12%	12%	13%

Table .8: The annual official economic variables for the period of 2015-2021

Regressor/year	2015	2016	2017	2018	2019	2020	2021
CE	1735.1	1581.1	1651.4	2046.1	2079.1	1877.1	2198
CP	102.01	104.31	108.81	110.71	112.51	114.61	118.31
Export	52.59	37.3	49.49	65.36	67.38	35.09	47.34
Import	53670	35404	44037	52158	55564	50803	52068
FDI	3148.5	1634.8	3032.8	2697.1	2417.5	2203.1	4491.1
IPOD	754.2	842.4	832.5	832.7	1169.1	1025.1	1106.8
IR	9.00%	15.70%	16.50%	12.10%	11.40%	13.20%	16.90%
MD	15697.3	17641.4	19321.8	21492.2	23523.6	28158.7	31811.6
MPR	192.44	253.49	305.79	306.08	306.92	358.81	411.98
MS	18743490.4	20908816.1	22891875.4	25543187.3	27634257.4	30596591.2	34097224.7
NIFA	12807.75	8698.89	11492.58	18323.73	14670.96	15772.71	15896.43
RE	110.80	116.20	105.40	108.10	124.20	119.50	117.10
SE	28642.25	26874.62	38243.19	31430.51	26842.07	40270.72	42716.44
BOT	-2641.1	-4463.1	2911.5	2569.8	2248.1	-5053.1	8938.3
INT.R	11%	14%	14%	14.00%	13.50%	12.50%	11.50%