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A practical approach to fixed point theory in partial-metric spaces using simulation functions

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Abstract. This paper investigates coincidence point results for self-mappings in partial-metric spaces via simulation functions. By introducing a generalized contraction condition involving a simulation function and an auxiliary mapping H, we establish sufficient conditions for the existence and uniqueness of coincidence points and common fixed points. Our approach not only unifies several existing fixed point theorems in the literature but also provides a genuine extension by weakening conventional contraction assumptions. The theoretical findings are illustrated by a concrete example in a non-standard partial-metric space setting, confirming the applicability and effectiveness of the proposed framework. As a special case, our results recover and generalize recent fixed point theorems in both metric and partial-metric spaces.

Keywords: Partial-metric space, generalized contraction, simulation function.
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1 Introduction

Fixed point theory constitutes one of the most dynamic and applicable branches of nonlinear analysis, with its foundations rooted in Banach's seminal contraction principle from 1922. Over the decades, this theory has been extensively generalized and refined through various approaches among which the introduction of *simulation functions* by Karapinar et al. [1,7,9] represents a particularly elegant and powerful tool. Simulation functions allow the formulation of weakened contractive conditions, thereby extending the classical Banach theorem to a wider class of mappings while preserving the essential properties of existence and uniqueness.

Parallel to these developments, the framework of *partial-metric spaces*, introduced as a generalization of metric spaces, has attracted considerable attention. In partial-metric spaces, the self-distance of a point need not be zero, which makes them suitable for modeling problems in computer science, domain theory, and quantitative semantics where distance may carry a non-zero intrinsic weight. Numerous examples and applications of partial-metric spaces are documented in the literature; see, e.g., [5,6]. This flexibility makes partial-metric spaces a natural setting in which to explore fixed point results under relaxed assumptions.

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Motivated by the interplay between simulation functions and generalized metric structures, the present paper aims to establish new coincidence and common fixed point theorems for self-mappings in partial-metric spaces. Specifically, we introduce a generalized contraction condition involving a simulation function ζ and an auxiliary mapping H, which substantially weakens the usual contractive hypotheses and thereby covers a broader spectrum of mappings. Our main results not only unify several known theorems in metric and partial-metric spaces but also offer genuine extensions that highlight the unifying role of simulation functions.

The paper is organized as follows. Section 2 collects the necessary definitions and preliminary results on partial-metric spaces, simulation functions, C_G-simulation functions, and compatibility notions. In Section 3 we state and prove our main theorems, providing detailed arguments based on Picard–Jungck sequences and various completeness/compatibility assumptions. An illustrative example is given to demonstrate the applicability of the results. Finally, Section 4 summarizes the contributions of the work and indicates possible directions for future research.

2 Preliminaries

The following definitions and preliminaries are required to establish the main results.

Definition 2.1. [5] On a non-empty set X, a function $d : X \times X \to [0, +\infty)$ is called a partial-metric if it satisfies the following conditions for all $v, \omega, z \in X$:

(ρ_1) $v = \omega$ iff $d(v, v) = d(v, \omega) = d(\omega, \omega)$;

(ρ_2) $d(v, v) \leq d(v, \omega)$;

(ρ_3) $d(v, \omega) = d(\omega, v)$;

(ρ_4) $d(v, \omega) \leq d(v, z) + d(z, \omega) - d(z, z)$.

The pair (X, d) is called a partial-metric space.

Definition 2.2. [5] If (X, d) is a partial-metric space and $\{v_m\}$ is a sequence in X, then:

(P_1) $\{v_m\}$ is convergent to a limit $v \in X$, if

$$\lim_{m \to +\infty} d(v_m, v) = d(v, v).$$

(P_2) $\{v_m\}$ is a Cauchy sequence if

$$\lim_{m,q \to +\infty} d(v_m, v_q) \text{ exists and is finite.}$$

Moreover, we say that the partial-metric space (X, d) is complete if every Cauchy sequence $\{v_m\}$ in X converges to a point $v \in X$, that is

$$\lim_{m,q \to +\infty} d(v_m, v_q) = \lim_{m \to +\infty} d(v_m, v) = d(v, v).$$

Definition 2.3. [4] A simulation function is a function $\zeta : [0, +\infty) \times [0, +\infty) \to \mathbb{R}$ satisfying the following conditions:

(ζ_1) $\zeta(0, 0) = 0$;

(ζ_2) $\zeta(t, s) < s - t$ for all $t, s > 0$;

(ζ_3) if $\{t_n\}, \{s_n\}$ are sequences in $(0, +\infty)$ such that $\lim_{n \to +\infty} t_n = \lim_{n \to +\infty} s_n > 0$, then

$$\limsup_{n \to +\infty} \zeta(t_n, s_n) < 0.$$

Let $\mathcal{Z}$ denote the family of all simulation functions $\zeta : [0, +\infty) \times [0, +\infty) \to \mathbb{R}$. Due to the axiom (ζ_2), we have $\zeta(t, t) < 0$ for all $t > 0$.

Example 2.4. [1] Let $\phi_i : [0, +\infty) \to [0, +\infty)$ be continuous functions with $\phi_i(t) = 0$ iff $t = 0$, for $i = 1, 2$. We define the mapping $\zeta_1 : [0, +\infty) \times [0, +\infty) \to \mathbb{R}$ by

$$\zeta_1(t, s) = \phi_1(s) - \phi_2(t) \text{ for all } t, s \in [0, +\infty),$$

where $\phi_1(t) < t \leq \phi_2(t)$ for all $t > 0$.

Then ζ_1 is a simulation function.

Example 2.5. [4] Let $f, g : [0, +\infty) \times [0, +\infty) \to (0, +\infty)$ be two continuous functions with respect to each variable such that $f(t, s) > g(t, s)$ for all $t, s > 0$. We define the mapping $\zeta_2 : [0, +\infty) \times [0, +\infty) \to \mathbb{R}$ by

$$\zeta_2(t, s) = s - \frac{f(t, s)}{g(t, s)} t \text{ for all } t, s \in [0, +\infty),$$

which is also a simulation function.

Definition 2.6. [2] A function $G : [0, +\infty) \times [0, +\infty) \to \mathbb{R}$ has the property C_G, if there exists a $C_G \geq 0$ such that

(1) $G(s, t) > C_G$ implies $s > t$;

(2) $G(t, t) \leq C_G$ for all $t \in [0, +\infty)$.

Definition 2.7. [2] A C_G-simulation function is a function $\zeta : [0, +\infty) \times [0, +\infty) \to \mathbb{R}$ satisfying the following conditions:

(a) For all $t, s > 0$ we have $\zeta(t, s) < G(s, t)$, where $G : [0, +\infty) \times [0, +\infty) \to \mathbb{R}$ is a function that has the property C_G;

(b) If $\{t_n\}$ and $\{s_n\}$ are sequences in $(0, +\infty)$ such that

$$\lim_{n \to +\infty} t_n = \lim_{n \to +\infty} s_n > 0,$$

then

$$\limsup_{n \to +\infty} \zeta(t_n, s_n) < C_G.$$

Let $\mathcal{Z}_G$ be the family of all C_G-simulation functions $\zeta : [0, +\infty) \times [0, +\infty) \to \mathbb{R}$.

Remark 2.8. Each simulation function in the sense of Definition 2.3 is also a C_G-simulation function as in Definition 2.7, but the converse is not true. Consider the example of the function $G(s, t) = s - t$ (see [1] for details).

Lemma 2.9. *[5] Let (X, d) be a partial-metric space and $\{v_m\}$ be a sequence in X such that*

$$\lim_{m \to +\infty} d(v_m, v_{m+1}) = 0.$$

If $\lim_{m,q \to +\infty} d(v_m, v_q) \neq 0$, then there exist $\epsilon > 0$ and two sub-sequences $\{v_{m_k}\}, \{v_{q_k}\}$ of $\{v_m\}$ such that

$$\begin{aligned}\lim_{k \to +\infty} d(v_{m_k}, v_{q_k}) &= \lim_{k \to +\infty} d(v_{m_k}, v_{q_k+1}) = \lim_{k \to +\infty} d(v_{m_k+1}, v_{q_k}) \\ &= \lim_{k \to +\infty} d(v_{m_k+1}, v_{q_k+1}) = \epsilon.\end{aligned}$$

For the following, let f and g be self-mappings of a partial-metric space (X, d).

Definition 2.10. [1] If $w = fx = gx$ for some $x \in X$, then x is called a coincidence point of f and g, and w is called a point of coincidence of f and g.

If also $x = w$, then x is a common fixed point of f and g.

Definition 2.11. [8] Mappings f and g are called

(i) Compatible if, for every sequence $\{x_n\}$ in X such that $\{fx_n\}$ and $\{gx_n\}$ are convergent to some $t \in X$, then

$$\lim_{n\to+\infty} d\left(f(gx_n), g(fx_n)\right) = d(t,t).$$

(ii) Non-compatible if there exists at least one sequence $\{x_n\}$ in X such that $\{fx_n\}$ and $\{gx_n\}$ are convergent to some $t \in X$, but

$$\lim_{n\to+\infty} d\left(f(gx_n), g(fx_n)\right) \text{ does not exist.}$$

Definition 2.12. [11] The pair (f, g) is weakly compatible if f and g commute at their coincidence points, meaning for all $x \in X$ such that $w = fx = gx$ we have

$$g(fx) = f(gx).$$

Remark 2.13. [11] If a pair (f, g) is compatible, then it is also weakly compatible, but the converse is not true.

Definition 2.14. [2] A sequence $\{x_n\}_{n\in\mathbb{N}} \subseteq X$ is a Picard–Jungck sequence of the pair (f, g) (based on $x_0 \in X$) if $y_n = fx_n = gx_{n+1}$ for all $n \in \mathbb{N}$.

Remark 2.15. [10] If we have $f(X) \subset g(X)$ or $g(X) \subset f(X)$, then it is certain that a Picard–Jungck sequence exists for the pair (f, g), but the converse is not true.

Theorem 2.16. *[2] Let f and g be weakly compatible self-mappings of a set X. If f and g have a unique point of coincidence $w = fx = gx$, then w is a unique common fixed point of f and g.*

Definition 2.17. [2] A mapping f is called a $(\mathcal{Z}_G, g)$-contraction if there exists $\zeta \in \mathcal{Z}_G$ such that

$$\zeta(d(fx, fy), d(gx, gy)) \geq C_G, \tag{2.1}$$

for all $x, y \in X$ with $gx \neq gy$.

In the case where $g = i_X$ (identity mapping on X) and $C_G = 0$, we get what is called a $\mathcal{Z}$-contraction (see [3]).

3 Main results

In this section, we establish some results on the existence and uniqueness of common fixed points using simulation functions in the framework of partial-metric spaces.

Let F denote the family of mappings $H : [0, +\infty) \to [0, +\infty)$ that satisfies the following conditions

$$0 < H(t) \leq t \quad \text{for all} \quad t \in (0, +\infty) \text{ and } H(0) = 0. \tag{3.1}$$

Theorem 3.1. *Let (X,d) be a partial-metric space, and let $f,g : X \to X$ be self-mappings. Assume that there exists $\zeta \in \mathcal{Z}_G$ and $H \in F$ such that*

$$\zeta(d(fx,fy),H(d(gx,gy))) \geq C_G, \tag{3.2}$$

for all $x,y \in X$ with $gx \neq gy$.

Assume further that there exists a Picard–Jungck sequence $\{x_n\}_{n\in\mathbb{N}}$ of (f,g). Suppose that at least one of the following conditions holds:

(i) $(g(X),d)$ is complete and f and g are weakly compatible.

(ii) $f(X) \subset g(X)$, $(f(X),d)$ is complete and f and g are weakly compatible.

(iii) (X,d) is complete, g is continuous and (f,g) is compatible.

Then f and g have a unique common fixed point.

Proof. First of all, we shall prove that the point of coincidence of f and g is unique (if it exists). Suppose that z_1 and z_2 are distinct points of coincidence of f and g. It follows that there exist two points v_1 and v_2 $(v_1 \neq v_2)$ such that $fv_1 = gv_1 = z_1$ and $fv_2 = gv_2 = z_2$. Then (3.2) implies

$$C_G \leq \zeta\left(d\left(fv_1,fv_2\right),H(d\left(gv_1,gv_2\right))\right) = \zeta\left(d\left(z_1,z_2\right),H(d\left(z_1,z_2\right))\right) < G\left(H(d\left(z_1,z_2\right)),d\left(z_1,z_2\right)\right).$$

According to Definition 2.6, this would mean that

$$H(d\left(z_1,z_2\right)) > d\left(z_1,z_2\right),$$

which contradicts the defining properties of H, see (3.1).

In order to prove that f and g have a point of coincidence, we take the Picard–Jungck sequence $\{x_n\}_{n\in\mathbb{N}}$. According to Definition 2.14 we have $y_n = fx_n = gx_{n+1}$ for all $n \in \mathbb{N}$.

If $y_k = y_{k+1}$ for some $k \in \mathbb{N}$, then $gx_{k+1} = y_k = y_{k+1} = fx_{k+1}$ and f and g have a point of coincidence. Therefore, suppose that $y_n \neq y_{n+1}$ for all $n \in \mathbb{N}$. Substituting $x = x_{n+1}$ and $y = x_{n+2}$ in (3.2) we obtain that

$$\begin{aligned} C_G \leq & \zeta\left(d\left(fx_{n+1},fx_{n+2}\right),H(d\left(gx_{n+1},gx_{n+2}\right))\right) \\ = & \zeta\left(d\left(y_{n+1},y_{n+2}\right),H(d\left(y_n,y_{n+1}\right))\right) \\ < & G\left(H(d\left(y_n,y_{n+1}\right)),d\left(y_{n+1},y_{n+2}\right)\right). \end{aligned}$$

Using Definition 2.6, we have

$$H(d\left(y_n,y_{n+1}\right)) > d\left(y_{n+1},y_{n+2}\right).$$

Hence, for all $n \in \mathbb{N}$ we get

$$d\left(y_{n+1},y_{n+2}\right) < d\left(y_n,y_{n+1}\right).$$

Therefore there exists $D \geq 0$ such that

$$\lim_{n\to+\infty} d\left(y_n,y_{n+1}\right) = D \geq 0.$$

Suppose that $D > 0$. Since

$$d\left(y_{n+1},y_{n+2}\right) < H(d\left(y_n,y_{n+1}\right)) < d\left(y_n,y_{n+1}\right),$$

and both $d\left(y_{n+1}, y_{n+2}\right)$ and $d\left(y_n, y_{n+1}\right)$ tend to D as $n \to +\infty$, that would mean that $H(d\left(y_n, y_{n+1}\right))$ also tends to D as $n \to +\infty$.

Using (b) of Definition 2.7, we get

$$C_G \leq \limsup_{n\to+\infty} \zeta\left(d\left(y_{n+1}, y_{n+2}\right), H(d\left(y_n, y_{n+1}\right))\right) < C_G,$$

which is a contradiction, hence

$$\lim_{n\to+\infty} d\left(y_n, y_{n+1}\right) = D = 0. \tag{3.3}$$

We next show that $y_n \neq y_m$ for $n \neq m$.

Indeed, suppose that $y_n = y_m$ for some $n > m$. By the definition of the Picard–Jungck sequence this would mean that

$$x_{m+1} \in g^{-1}(y_m),$$

and

$$x_{n+1} \in g^{-1}(y_n) = g^{-1}(y_m).$$

Then it is possible to choose $x_{n+1} = x_{m+1}$ and also $y_{n+1} = y_{m+1}$. Following the previous arguments, we have

$$d\left(y_n, y_{n+1}\right) < d\left(y_{n-1}, y_n\right) < \cdots < d\left(y_m, y_{m+1}\right) = d\left(y_n, y_{n+1}\right),$$

thus

$$d\left(y_n, y_{n+1}\right) < d\left(y_n, y_{n+1}\right),$$

which is a contradiction.

Now, we show that $\{y_n\}$ is a Cauchy sequence. Suppose, on the contrary, that it is not. Then

$$\lim_{n,q\to+\infty} d\left(y_n, y_q\right) \neq 0.$$

According to (3.3) and Lemma 2.9, this would imply that there exist $\epsilon > 0$ and two subsequences $\{x_{m_k}\}, \{x_{n_k}\}$ of $\{x_n\}$ such that

$$\lim_{k\to+\infty} d(x_{n_k}, x_{m_k}) = \lim_{k\to+\infty} d(x_{n_k+1}, x_{m_k+1}) = \varepsilon > 0. \tag{3.4}$$

Putting $x = x_{m_k+1}$ and $y = x_{n_k+1}$ in (3.2) and using Definition 2.7, we obtain

$$\begin{aligned} C_G &\leq \zeta(d(fx_{m_k+1}, fx_{n_k+1}), H(d(gx_{m_k+1}, gx_{n_k+1}))) \\ &= \zeta(d(y_{m_k+1}, y_{n_k+1}), H(d(y_{m_k}, y_{n_k}))) \\ &< G(H(d(y_{m_k}, y_{n_k})), d(y_{m_k+1}, y_{n_k+1})). \end{aligned} \tag{3.5}$$

Therefore, using (3.5), (3.4) and (b) from Definition 2.7, we have

$$C_G \leq \limsup_{n\to+\infty} \zeta\left(d\left(y_{m_k+1}, y_{n_k+1}\right), d\left(y_{m_k}, y_{n_k}\right)\right) < C_G,$$

which is a contradiction, so

$$\lim_{n,q\to+\infty} d\left(y_n, y_q\right) = 0.$$

Therefore, the sequence $\{y_n\}$ is a Cauchy sequence.

Suppose that (i) holds, i.e., $(g(X), d)$ is complete. Then there exists $v \in X$ such that $gx_n \to gv$ as $n \to +\infty$, thus

$$\lim_{n,q\to+\infty} d\left(gx_n, gx_q\right) = \lim_{n\to+\infty} d\left(gx_n, gv\right) = 0. \tag{3.6}$$

We now prove that $fv = gv$.

Without loss of generality, we may assume that $y_n \neq gv$ for all $n \in \mathbb{N}$. Therefore, by (3.2) we have

$$C_G \leq \zeta\left(d\left(fx_n, fv\right), H(d\left(gx_n, gv\right))\right) < G\left(H(d\left(gx_n, gv\right)), d\left(fx_n, fv\right)\right) \text{ for all } n \in \mathbb{N}.$$

Using Definition 2.6, we get

$$d\left(fx_n, fv\right) < H(d\left(gx_n, gv\right)) < d\left(gx_n, gv\right) \to 0 \text{ as } n \to +\infty.$$

It implies that $y_n = fx_n \to fv$ as $n \to +\infty$, hence, $fv = gv$ is a unique point of coincidence of f and g.

Further, since f and g are weakly compatible, then according to Theorem 2.16 the pair (f, g) has a unique common fixed point.

Now, suppose that (ii) holds. We will use the same proof as condition (i) to conclude that the sequence $\{y_n\}$ is a Cauchy sequence, and since $(f(X), d)$ is complete, this means that there exists $u \in X$ such that $fx_n \to fu$ as $n \to +\infty$, this implies that

$$fx_{n-1} \to fu \text{ as } n \to +\infty,$$

therefore

$$gx_n \to fu \text{ as } n \to +\infty.$$

Since $f(X) \subset g(X)$, there exists $v \in X$ such that

$$gx_n \to gv \quad \text{as } n \to \infty.$$

Consequently, we again arrive at (3.6). Using an argument similar to that employed in case (i), we conclude that $fv = gv$, and hence v is the unique coincidence point of f and g.

And since f and g are weakly compatible, according to Theorem 2.16 they have a unique common fixed point.

Finally, suppose that (iii) holds. Since (X, d) is complete, there exists $v \in X$ such that $fx_n \to v$ as $n \to +\infty$, and

$$\lim_{n,q\to+\infty} d\left(fx_n, fx_q\right) = \lim_{n\to+\infty} d\left(fx_n, v\right) = d\left(v, v\right) = 0.$$

As g is continuous, $g\left(fx_n\right) \to gv$ as $n \to +\infty$. Considering (3.2) we get

$$C_G \leq \zeta\left(d\left(f\left(gx_n\right), fv\right), H(d\left(g\left(gx_n\right), gv\right))\right) < G\left(H(d\left(g\left(gx_n\right), gv\right)), d\left(f\left(gx_n\right), fv\right)\right).$$

Using Definition 2.6, we have

$$H(d\left(g\left(gx_n\right), gv\right)) > d\left(f\left(gx_n\right), fv\right),$$

by continuity of g this means

$$d\left(f\left(gx_n\right), fv\right) < d\left(g\left(gx_n\right), gv\right) = d\left(g\left(fx_{n-1}\right), gv\right) \to d\left(gv, gv\right) \text{ as } n \to +\infty. \tag{3.7}$$

As f and g are compatible, we have

$$\begin{aligned} d(fv, gv) &\leq d\left(f\left(gx_n\right), fv\right) + d\left(f\left(gx_n\right), g\left(fx_n\right)\right) + d\left(g\left(fx_n\right), gv\right) \\ &\quad - d\left(f\left(gx_n\right), f\left(gx_n\right)\right) - d\left(g\left(fx_n\right), g\left(fx_n\right)\right) \\ &\leq d\left(g\left(fx_{n-1}\right), gv\right) + d\left(f\left(gx_n\right), g\left(fx_n\right)\right) + d\left(g\left(fx_n\right), gv\right), \end{aligned}$$

and we have

$$d\left(f\left(gx_n\right), f\left(gx_n\right)\right) \leq 2d\left(f\left(gx_n\right), g\left(fx_n\right)\right) - d\left(g\left(fx_n\right), g\left(fx_n\right)\right),$$

thus

$$d\left(f\left(gx_n\right), f\left(gx_n\right)\right) + d\left(g\left(fx_n\right), g\left(fx_n\right)\right) \leq 2d\left(f\left(gx_n\right), g\left(fx_n\right)\right), \tag{3.8}$$

and according to Definition 2.11

$$\lim_{n\to+\infty} d\left(f\left(gx_n\right), g\left(fx_n\right)\right) = d\left(v, v\right) = 0. \tag{3.9}$$

From (3.8), (3.9) and the continuity of g we conclude

$$d\left(gv, gv\right) = \lim_{n\to+\infty} d\left(g\left(fx_n\right), g\left(fx_n\right)\right) = 0. \tag{3.10}$$

Then, according to (3.7), (3.9) and (3.10) we get

$$d\left(g\left(fx_{n-1}\right), gv\right) + d\left(f\left(gx_n\right), g\left(fx_n\right)\right) + d\left(g\left(fx_n\right), gv\right) \to 0 + 0 + 0 = 0 \text{ as } n \to +\infty.$$

Hence, $d(fv, gv) = 0$, that is, the mappings f and g have a unique point of coincidence.

Since f and g are compatible, they are also weakly compatible, so according to Theorem 2.16, f and g have a unique common fixed point. This completes the proof. □

Remark 3.2. Consider the case where, for each $t \in X$ we have $d(t, t) = 0$.

The main result in [2] is obtained if we consider $H = i_{[0,+\infty)}$ in Theorem 3.1.

Similarly, Theorem 3.1 yields the main result of [10] by choosing $C_G = 0$.

Theorem 3.3. *Let us consider the same hypotheses as Theorem 3.1. Suppose that at least one of the following conditions holds:*

(H1) $(g(X), d)$ is complete,

(H2) $f(X) \subset g(X)$ and $(f(X), d)$ is complete.

Then f and g have a unique point of coincidence.

Proof. The proof of this theorem follows directly from the proof of Theorem 3.1 under conditions (i) and (ii). However, this theorem does not require the compatibility or weak compatibility of the pair (f, g). □

Corollary 3.4. *Both Theorems 3.1 and 3.3 could be used to show the existence and uniqueness of the fixed point of a single function f, if we consider that $g = i_X$. It is sufficient to prove that the pair (f, g) has a unique point of coincidence to conclude the existence and uniqueness of the fixed point.*

Example 3.5. Let $I([0, +\infty))$ denote the set of all intervals $[a, b]$ with $0 < a \leq b$. Let $d : I \times I \to [0, +\infty)$ be the function such that

$$d([a, b], [c, d]) = \max(b, d) - \min(a, c).$$

Then $(I([0, +\infty)), d)$ is a partial-metric space, because for all $[a, b], [c, d], [e, f] \in I([0, +\infty))$, the function d satisfies

- $d([a,b],[a,b]) = d([a,b],[c,d]) = d([c,d],[c,d]) \iff [a,b] = [c,d]$,
- $d([a,b],[a,b]) \leq d([a,b],[c,d])$,
- $d([a,b],[c,d]) = d([c,d],[a,b])$,
- $d([a,b],[c,d]) \leq d([a,b],[e,f]) + d([e,f],[c,d]) - d([e,f],[e,f])$.

The self-distance $d([a,b],[a,b])$ is $b-a$, the length of the interval $[a,b]$. Define the mappings $f,g : [0,+\infty) \to [0,+\infty)$ by

$$fx = x + \frac{1}{2}, \quad gx = 4x + e^{2x}.$$

In order to solve the nonlinear equation

$$f([a,b]) = g([a,b]).$$

Theorem 3.3 can be applied using the following simulation function

$$\zeta(t,s) = \frac{9}{10}(s-t) \text{ for all } s,t \in [0,+\infty),$$

the function $G : [0,+\infty) \times [0,+\infty) \to \mathbb{R}$ defined by

$$G(s,t) = s - t \text{ for all } s,t \in [0,+\infty),$$

with the constant $C_G = 0$, and the mapping H defined as follows

$$H(t) = \ln(t+1) \text{ for all } t \in [0,+\infty).$$

Since f and g are both increasing functions we have that

$$\begin{aligned}
&\zeta(d(f([a,b]),f([a',b'])),H(d(g([a,b]),g([a',b'])))) \\
&\quad = \frac{9}{10}\big(H(d(g([a,b]),g([a',b']))) - d(f([a,b]),f([a',b']))\big) \\
&\quad = \frac{9}{10}\big(H(\max(gb,gb') - \min(ga,ga')) - \max(fb,fb') + \min(fa,fa')\big) \\
&\quad = \frac{9}{10}\big(H(g(\max(b,b')) - g(\min(a,a'))) - f(\max(b,b')) + f(\min(a,a'))\big) \\
&\quad = \frac{9}{10}\Big(\ln(4(\max(b,b')) + e^{2\max(b,b')} - 4(\min(a,a')) - e^{2\min(a,a')} + 1) - \max(b,b') + \min(a,a')\Big),
\end{aligned}$$

for all $[a,b],[a',b'] \in I([0,+\infty))$. Now, we prove that

$$\frac{9}{10}\Big(\ln(4(\max(b,b') - \min(a,a')) + e^{2\max(b,b')} - e^{2\min(a,a')} + 1) - \max(b,b') + \min(a,a')\Big) \geq 0.$$

Set $M = \max(b,b')$ and $m = \min(a,a')$. Since $M \geq m \geq 0$, we have

$$e^{2M} - e^{2m} = e^{2m}(e^{2(M-m)} - 1) \geq e^{2(M-m)} - 1.$$

Thus

$$\ln\big(4(M-m) + e^{2M} - e^{2m} + 1\big) \geq \ln\big(4(M-m) + e^{2(M-m)}\big).$$

Define $hx = \ln(4x + e^{2x}) - x$ for $x \geq 0$. A direct computation shows that $h'x \geq 0$, hence h is increasing and $h(0) = 0$. Therefore

$$\ln(4x + e^{2x}) \geq x \quad \text{for all } x \geq 0.$$

Applying this with $x = M - m$ yields

$$\ln\big(4(M-m) + e^{2M} - e^{2m} + 1\big) \geq M - m.$$

Multiplication by $\frac{9}{10}$ completes the proof.

Thus, ζ satisfies (3.2). Since $g(I([0, +\infty))) = I([1, +\infty))$, Theorem 3.3 under condition (i) implies that f and g have a unique point of coincidence.

4 Conclusion

This work has presented new fixed point theorems for self-mappings in partial-metric spaces through the lens of simulation functions. By introducing a generalized contraction condition that incorporates both a simulation function $\zeta \in \mathcal{Z}_G$ and an auxiliary mapping $H \in F$, we have established sufficient criteria for the existence and uniqueness of coincidence points and common fixed points.

The main contributions of this paper can be summarized as follows:

- A unified framework that extends and generalizes several existing fixed point results in both metric and partial-metric spaces.
- The use of simulation functions to weaken traditional contractive conditions, thereby covering a broader class of mappings.
- The introduction of a Picard–Jungck sequence technique to prove the existence of coincidence points under various completeness and compatibility assumptions.
- An illustrative example that validates the applicability of the proposed theorems in a nonstandard partial-metric space setting, demonstrating the practical relevance of our theoretical findings.

Our results not only enrich the fixed point theory in generalized metric spaces but also offer a flexible tool for researchers working in nonlinear analysis, computational mathematics, and related fields. Future research may explore further extensions, such as applying these results to coupled or tripled fixed point problems, or adapting the framework to other generalized metric structures such as b-metric, G-metric, or modular metric spaces.

Declarations

Conflict of interest

The authors declare no conflict of interest.

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